成功大學物理研究所博士班 97 學年度[資格考試] **電動力學**試題

Electrodynamics (Part I, 50 points) 2008-10-16

1. A static charge distribution produces a radial electric field $\vec{E}(r) = A\frac{e^{-br}}{r^2}\hat{r}$, where *A* and *b* are constants.

(a) Find the charge density. (10 points)
(b) Find the total charge. (5 points)

2. Two infinite plane conductors at zero potential are parallel and separated by a distance *d*. A pint charge *q* is brought to the middle of these planes.

(a) Find the electrostatic potential at the location of the point charge due to induced surface charge on the planes. (10 points)
(b) Find the electric potential energy of this arrangement. (5 points)

3. A surface charge density $\sigma(\theta) = \sigma_0 \cos\theta$ is glued to the surface of a spherical shell of radius *R*, where σ_0 is a constant and θ is the polar angle.

(a) Find the electrostatic potential inside and outside of the spherical shell. (10 points)
(b) Find the electric field inside and outside of the spherical shell. (10 points)

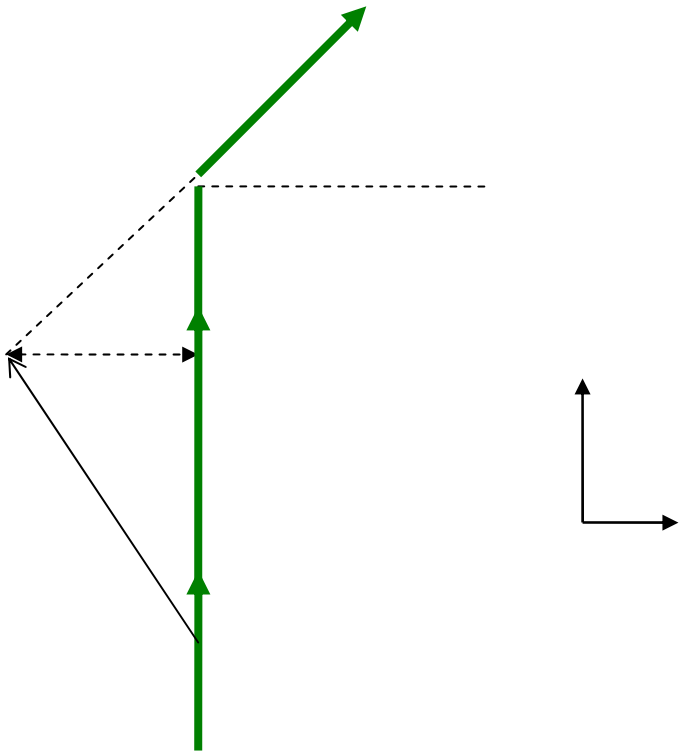
Electrodynamics (Part II, 50 points) 2008-10-16

1. An infinite wire carrying a steady current *I* is bent at point A as show in the Figure.

(a) What are the magnitude and direction of magnetic field $\boldsymbol{B}_p$ at point P, which is at a distance *s* from the straight segment of the wire, as shown in the Figure. (10 points)

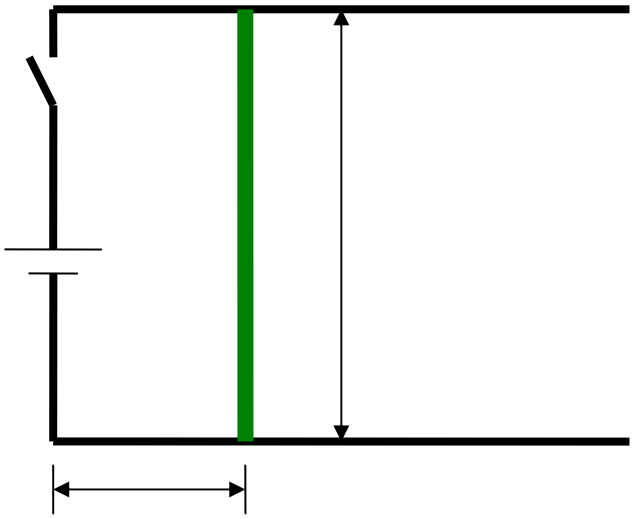
(b) Find the torque $\boldsymbol{N}_p$ acting on the magnetic dipole $\vec{m}$ located at P. Here the magnetic dipole is expressed as a three-component vector $\vec{m} = m_x\hat{i} + m_y\hat{j} + m_z\hat{k}$ with magnitude $m = |\vec{m}| = \sqrt{m_x^2 + m_y^2 + m_z^2}$. (5 points)

(c) For what angle ϕ between $\vec{m}$ and $\boldsymbol{B}_p$ is the dipole's energy at its lowest value? What is this lowest dipole's energy? What is the magnitude of torque when $\vec{m}$ experiences in this lowest energy configuration? (10 points)



2. Consider a track shown below, consisting of two parallel, resistanceless, conducting rails separated by a distance w. Assume that the track is very long. A rod of resistance R and mass m slides along the frictionless rails. The track is horizontal and three is a constant uniform magnetic field B_o directed vertically downward in the whole region of the Figure. At time t=0 the rod is located at x=0 and motionless. The battery has a voltage V_o with polarity as shown. At t=0 the switch, S, is closed.

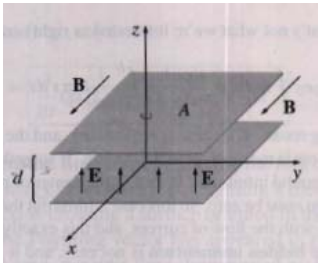
(a) What is the maximum velocity achieved by the rod? (10 points)
(b) Express the velocity as a function of time in terms of w, R, m, B_o, and V_o. (15 points)



Electrodynamics (Part III, 50 points) 2008-10-16

1. A large parallel-plate capacitor (with uniform electric filed $\boldsymbol{E}=E\hat{z}$) is placed in a uniform magnetic field $\boldsymbol{B}=B\hat{x}$, as shown in Fig. 1.
   (a) Find the electromagnetic momentum in the space between the plates. (10 points)

Fig. 1



   (b) Now a resistive wire is connected between the plates, along the *z* axis, so that the capacitor slowly discharges. The current through the wire will experience a magnetic force; what is the total impulse delivered to the system. (10 points)

2. Picture the electron as a uniformly charged spherical shell, with charge *e* and radius *R*, spinning at angular velocity ω.
   (a) Calculate the total energy contained in the electromagnetic fields. (10 points)
   (b) Calculate the total angular momentum contained in the fields. (10 points)

3. A point charge *q* is a distance $a > R$ from the axis of an infinite solenoid (radius *R*, *n* turns per unit length, current *I*). Find the linear momentum and the angular momentum in the fields. (10 points)