電動力學（Ⅱ）

（Ⅰ）由 Biot－Savart 定律

$$\vec{B}(\vec{x})=\frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{x}')*(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}d^3x'$$

其中 $\vec{J}$ 爲電流密度

(a) 證明 $\vec{\nabla}\bullet\vec{B}=0$

(b) 證明 $\vec{\nabla}\times\vec{B}=\mu_0\vec{J}$

(c) 如果 $\vec{B}=\vec{\nabla}\times\vec{A}$ 請寫 T. $\vec{A}$ 與 $\vec{J}$ 的關係式

（Ⅱ）

(a) 已知 $\vec{B}(\vec{x},t)=B_0\sin\omega t\,\hat{i}$ (B_0 爲常數) 求空間中的電場分佈 $\vec{E}(\vec{x},t)$

(b) 由(a)的結果求對應的電流密度 $\vec{J}(\vec{x},t)$

(c) 求對應的電流密度 $\rho(\vec{x},t)$

(d) 如果 $\vec{B}(\vec{x},t)=B_0(x)\ \hat{i}\sin\omega t$ 這個 $\vec{B}$ 有沒有意義？如果有的話則對應的

$\vec{E}(\vec{x},t)=?$

Electrodynamics Qualify Exam (Part I, 50 points) 2011-03-31

1. A static charge distribution produces a radial electric field $\vec{E}(r)=C\frac{e^{-Dr}}{r^2}\hat{r}$, where C and D are constants.

a) Find the charge density. (10 points)
b) Find the total charge. (5 points)

2. The time-averaged potential of a neutral hydrogen atom is given by

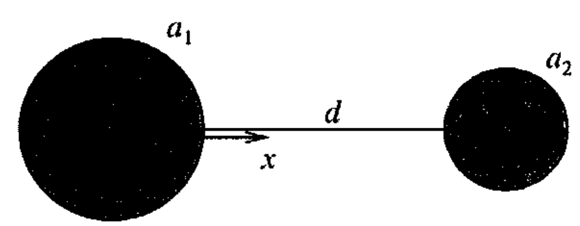
$$\Phi(r)=\frac{q}{4\pi\varepsilon_0}\frac{e^{-Dr}}{r}(1+\frac{Dr}{2})$$

where q is the magnitude of the electronic charge, and $D^{-1}=a_0/2$, a_0 being the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your result physically. (20 points)

3. Two long, cylindrical conductors of radii a_1 and a_2 are parallel and separated by a distance d, which is large compared with either radius. Show that the capacitance per unit length is given approximately by

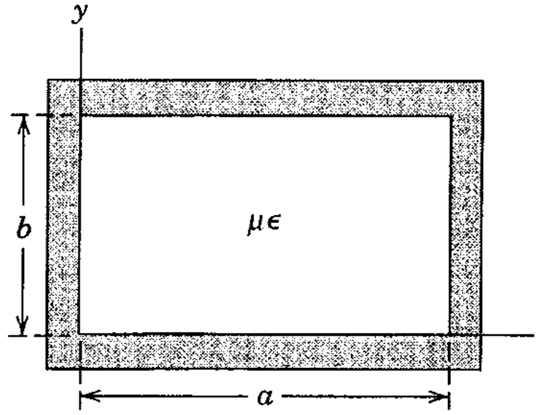
$$C=\pi\varepsilon_0\left(\ln\frac{d}{a}\right)^{-1}$$

where a is the geometrical mean of the two radii. (15 points)

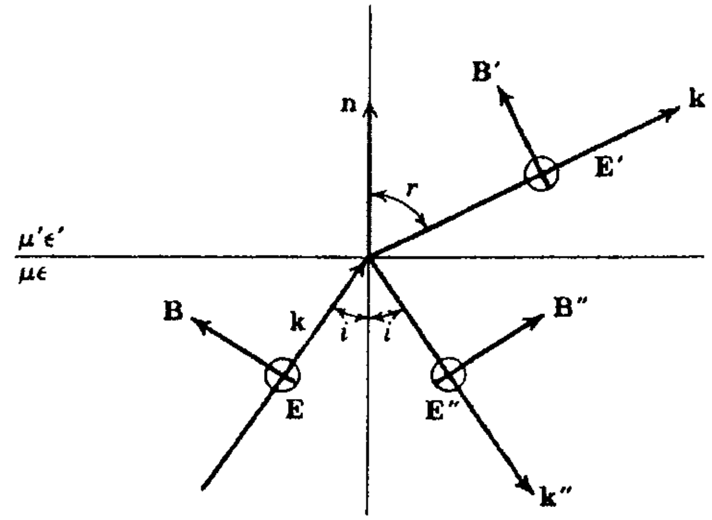


(c) Can you find a formula in terms of n, n', Z, Z' for a special angle at which the reflected wave vanishes? (5pts)

2. (a) Please explain the physical meaning of the *Transverse Magnetic* (TM) wave and the *Transverse Electric* (TE) wave. (8 pts) (b) Consider the propagation of TE waves in a rectangular waveguide with dimensions a, b, as shown in the figure. Assume the wall is a perfect conductor, find the solutions for the eigenfunction modes (10 pts)
(c) Find the lowest cutoff frequency in (b). (7 pts)



Electrodynamics Part III (50 points)



1. In a media plane waves propagate at a speed $v = 1/\sqrt{\quad} = c/n$, where n is the refraction index and $c = 1/\sqrt{\quad_0 \quad_0}$ is a fundamental constant. Consider a plane electromagnetic wave with wave number k which strikes a plane interface between different media. To satisfy the boundary conditions at the interface there must be a reflected and refracted waves with respective wave numbers k'', k' and amplitudes which depend on the particular polarization. In each region the fields have the form $\mathbf{E} = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{x} - i \quad t}$, $\mathbf{H} = \mathbf{k} \times \mathbf{E}/Z$, where $Z = \sqrt{\quad / \quad}$ is the impedance ($Z_0 = \sqrt{\quad_0 / \quad_0} = 376.6$ ohms is a fundamental constant). In all cases, the angle of incidence equals the angle of reflection and Snell's law is satisfied.
Consider in particular the case where the wave is *linearly polarized* with the electric field **perpendicular** to the plane of incidence, as indicated in the figure.
(a) Using the appropriate boundary conditions, please find the amplitudes of both the refracted and the reflected electric field relative to the incident electric field amplitude in terms of the incident angle and n, n', Z, Z' (10 pts).
(b) Beyond a certain incident angle $i0$, the refracted angle determined from Snell's law is imaginary. Physically this is the phenomenon of *total internal reflection*. Please find a formula for $i0$ in terms of n, n', Z, Z'. (10 pts)